On periodic groups with given properties of finite subgroups

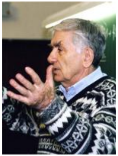
D. V. Lytkin

Siberian State University of Telecommunications
and Information Sciencies, Novosibirsk

Istanbul, 2011

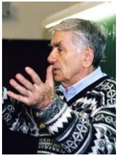
P. S. Novikov, S. I. Adian, 1968

In free periodic groups of exponent p where $p \geqslant 665$, every finite subgroup is cyclic.



P. S. Novikov, S. I. Adian, 1968

In free periodic groups of exponent p where $p \geqslant 665$, every finite subgroup is cyclic.



Main goal

Specify some properties of finite subgroups of a periodic group G which gaurantee local finiteness of G.

Definition (A. K. Shlöpkin)

Let $\mathcal{F}$ be some class of finite groups. We say that a periodic group G is saturated with groups from $\mathcal{F}$, if every finite subgroup $H \leqslant G$ is contained in a subgroup which is isomorphic to some group of $\mathcal{F}$.

Definition (A. K. Shlöpkin)

Let $\mathcal{F}$ be some class of finite groups. We say that a periodic group G is saturated with groups from $\mathcal{F}$, if every finite subgroup $H \leqslant G$ is contained in a subgroup which is isomorphic to some group of $\mathcal{F}$.

A. G. Rubashkin, K. A. Philippov, 2005

A periodic group saturated with finite simple groups $L_2(q)$, is isomorphic to a group $L_2(Q)$ for some locally finite field Q.

Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Suppose G is a periodic group all of whose finite subgroups of even order are contained in subgroups isomorphic to groups from $\mathfrak{N}$.

Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Suppose G is a periodic group all of whose finite subgroups of even order are contained in subgroups isomorphic to groups from $\mathfrak{N}$.

1. If G possesses an element of order 4 or a subgroup isomorphic to the alternating group of degree 4, then G is isomorphic to direct product of elementary abelian group of order at most 2^m and group $L_2(Q)$ for some locally finite field Q. In particular G is locally finite.

Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Suppose G is a periodic group all of whose finite subgroups of even order are contained in subgroups isomorphic to groups from $\mathfrak{N}$.

1. If G possesses an element of order 4 or a subgroup isomorphic to the alternating group of degree 4, then G is isomorphic to direct product of elementary abelian group of order at most 2^m and group $L_2(Q)$ for some locally finite field Q. In particular G is locally finite.

2. If $m \leqslant 1$ then either conclusion of item 1 of the theorem is true

Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Suppose G is a periodic group all of whose finite subgroups of even order are contained in subgroups isomorphic to groups from $\mathfrak{N}$.

1. If G possesses an element of order 4 or a subgroup isomorphic to the alternating group of degree 4, then G is isomorphic to direct product of elementary abelian group of order at most 2^m and group $L_2(Q)$ for some locally finite field Q. In particular G is locally finite.

2. If $m \leqslant 1$ then either conclusion of item 1 of the theorem is true or $m = 1$ and G is a non locally finite simple group whose Sylow 2-subgroup is elementary abelian, all involutions of G are conjugates and centralizer in G of any of them is isomorphic to direct product of a group of order 2 and a group $L_2(Q)$ where Q is an infinite locally finite field of charactreristic 2, whose multiplicative group does not possess elements of order 3.

Question 1.

Let V be a countable elementary abelian 2-group. Whether or not $\mathrm{Aut}(V)$ contains a subgroup H with the following properties:

a) H acts transitively on the set of involutions of V;

b) every finite subgroup of H fixes exactly one involution $v \in V$ and the stabilizer of v in H is isomorphic to the multiplicative group of some locally finite field of charactreristic 2?

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

(b) direct product of an elementary abelian 2-group and a cyclic 2-group;

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

(b) direct product of an elementary abelian 2-group and a cyclic 2-group;

(c) direct product of an elementary abelian 2-group and a group $C = \langle c_i, i = 1, 2, \dots \mid c_1^2 = 1, c_{i+1}^2 = c_i, i = 1, 2, \dots \rangle$;

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

(b) direct product of an elementary abelian 2-group and a cyclic 2-group;

(c) direct product of an elementary abelian 2-group and a group $C = \langle c_i, i = 1, 2, \dots \mid c_1^2 = 1, c_{i+1}^2 = c_i, i = 1, 2, \dots \rangle$;

(d) direct product of an elementary abelian 2-group and a dihedral 2-group;

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

(b) direct product of an elementary abelian 2-group and a cyclic 2-group;

(c) direct product of an elementary abelian 2-group and a group $C = \langle c_i, i = 1, 2, \dots \mid c_1^2 = 1, c_{i+1}^2 = c_i, i = 1, 2, \dots \rangle$;

(d) direct product of an elementary abelian 2-group and a dihedral 2-group;

(e) direct product of an elementary abelian 2-group and a group $D = \langle C, d \mid d^2 = 1, c_i^d = c_i^{-1} \rangle$.

Theorem 2.

Suppose that every finite subgroup of a 2-group T is isomorphic to a subgroup of direct product of a dihedral group and an elementary abelian group. Then T is isomorphic to one of the following groups:

(a) an elementary abelian 2-group;

(b) direct product of an elementary abelian 2-group and a cyclic 2-group;

(c) direct product of an elementary abelian 2-group and a group $C = \langle c_i, i = 1, 2, \ldots \mid c_1^2 = 1, c_{i+1}^2 = c_i, i = 1, 2, \ldots \rangle$;

(d) direct product of an elementary abelian 2-group and a dihedral 2-group;

(e) direct product of an elementary abelian 2-group and a group $D = \langle C, d \mid d^2 = 1, c_i^d = c_i^{-1} \rangle$.

In particular T is locally finite.

Theorem 3.

If all finite subgroups of a 2-group T are nilpotent of class 2 then T is nilpotent of class 2.

Theorem 3.

If all finite subgroups of a 2-group T are nilpotent of class 2 then T is nilpotent of class 2.

Corollary 1.

If all finite subgroups of a 2-group T are abelian then T is abelian.

Theorem 3.

If all finite subgroups of a 2-group T are nilpotent of class 2 then T is nilpotent of class 2.

Corollary 1.

If all finite subgroups of a 2-group T are abelian then T is abelian.

I. G. Lysenok, 1996

All finite subgroups of nilpotent Burnside group of exponent 2^n for $n \geqslant 13$ are embeddable into direct product of dihedral groups of order 2^{n+1}.

Question 2.

What is the largest number n which gaurantees nilpotency of every 2-group with finite subgroups of nilpotency class n?

Question 2.

What is the largest number n which gaurantees nilpotency of every 2-group with finite subgroups of nilpotency class n?

Question 3.

Is it true that a 2-group is nilpotent if every of its finite subgroups is nilpotent of class 3?

Corollary 2.

If conjugacy class orders in every finite subgroup of a 2-group T are at most 2 then the order of the derived subgroup of T is at most 2. In particular, T is of nilpotency class 2.

I. D. Macdonald

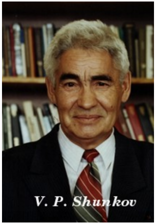
If G satisfies the identity $[x, y]^2 = 1$ then $G' = [G, G]$ is of exponent 4 and $G'' = [G', G']$ lies in the center of G.

I. D. Macdonald

If G satisfies the identity $[x, y]^2 = 1$ then $G' = [G, G]$ is of exponent 4 and $G'' = [G', G']$ lies in the center of G.

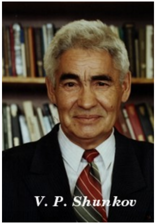
Theorem 4.

Suppose that in every finite subgroup of a 2-group T the identity $[x, y]^2 = 1$ holds. Then this identity holds also in a group T. In particular, T is locally finite, its derived subgroup is of exponent 4, and the second derived subgroup belongs to the center of T. Besides, if T is generated by involutions then its derived subgroup is elementary abelian.


V. P. Shunkov

Theorem 5.

If T is a Sylow 2-subgroup of a periodic group G not all of whose Sylow 2-subgroups are conjugates then, for every natural t, G possesses a Sylow 2-subgroup S not conjugate to T for which $|T \cap S| > t$.



Theorem 1.

Let m be a non-negative integer and $\mathfrak{N}$ a set of finite groups isomorphic to $E \times L$, where E is elementary abelian 2-group of order at most 2^m, and $L \simeq L_2(q)$ for some q.

Suppose G is a periodic group all of whose finite subgroups of even order are contained in subgroups isomorphic to groups from $\mathfrak{N}$.

1. If G possesses an element of order 4 or a subgroup isomorphic to the alternating group of degree 4, then G is isomorphic to direct product of elementary abelian group of order at most 2^m and group $L_2(Q)$ for some locally finite field Q. In particular G is locally finite.

2. If $m \leqslant 1$ then either conclusion of item 1 of the theorem is true or $m = 1$ and G is a non locally finite simple group Sylow 2-subgroup of which is elementary abelian, all involutions of G are conjugates and centralizer in G of any of them is isomorphic to direct product of a group of order 2 and a group $L_2(Q)$ where Q is an infinite locally finite field of charactreristic 2, whose multiplicative group does not possess elements of order 3.

Teşekkür edirim!

Bibliography

1. *Novikov P. S., Adian S. I.*// On infinite periodic groups, I-III, Mathematics of the USSR — Izvestiya. 1968. N 1. V. **32**. P. 212–244. N 2. V. **32**. P. 251–524. N 3. V. **32**. P. 709–731.
2. *Adian S. I.*// The Burnside problem and identities in groups (in Russian). 1975. M: Nauka.
3. *Slöpkin A. K.*// On some periodic groups saturated with finite simple groups. Siberian Advances in Mathematics. 1999. N 2. V. **9**. P. 100–108.
4. *Rubashkin A. G., Philippov K. A.*// On periodic groups saturated with groups $L_2(p^n)$. Siberian Math. J. 2005. N 6. V. **46**. P. 1119–1122.
5. *Lysenok I. G.*// Infinite Burnside groups of even exponent. Izvestya: Mathematics. 1996. N 3. V. **60**. P. 3–224.