In 1938 Philip Hall published the following result, which he described as "rather curious". Fix a prime $p$, and let $A(p)$ be the class of all finite abelian $p$-groups. Then

$$\sum_{G \mid n A(p)} \frac{1}{|G|} = \sum_{G \in A(p)} \frac{1}{|\text{Aut}(G)|}.$$  

We'll present related formulas involving families of non-abelian $p$-groups.